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# ESD.70J Engineering Economy

## Fall 2010 Session Two

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# Session two – Simulation

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- Objectives:
  - Generate random numbers
  - Get familiar with Monte Carlo simulation
  - Set up simulation using Data Table
  - Generate statistics from simulation
  - Draw histogram and cumulative distribution function (CDF)
    - Also called “target curve”

# Questions for “Big vs. Small”

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From the base case spreadsheet, we've calculated NPVs

However, we assumed deterministic demand forecasts for years 1, 2, and 3. This assumption is oversimplifying since actual demand will vary

⇒ Since life is uncertain, we want to simulate a range of possible NPV outcomes, the Min, Max, distributions, and the  $E[\text{NPV}]$ !

# Set up random generator

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Open ESD70session2-1.xls

# Excel's RAND() function

- Returns random number greater than or equal to 0 and less than 1, sampled from a uniform distribution
- To generate a random real number between a and b, use:  $=\text{RAND()}*(b-a)+a$
- In tab "RAND", the formula in cell C3:  
" $=\text{Entries!C9}*((1-\text{Entries!C25})+2*\text{Entries!C25}*\text{RAND()})$ "
  - Returns a uniformly distributed random demand for year 1 centered around 300, which may differ by plus or minus 50%
- Same logic applies for cell C4 and C5

# Random number generator

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Follow the instructions, step by step

1. Go to tab "RAND"
2. Type `"=Entries!C9*((1-Entries!C25)+2*Entries!C25*RAND())"` in cell C3
3. Type `"=Entries!C10*((1-Entries!C25)+2*Entries!C25*RAND())"` in cell D3
4. Type `"=Entries!C11*((1-Entries!C25)+2*Entries!C25*RAND())"` in cell E3
5. Press "F9" several times to see what happens

# Random number generator

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6. Click "Chart" under "Insert" menu
7. "Chart Type" select "XY(Scatter)", "Chart sub-type" select any one with lines, click "Next"
8. "Data Range" select B2:E3, click "Next"
9. "Chart options" select whatever pleases you, click "Next"
10. Choose "As object in" and click "Finish"
11. Press "F9" several times to see what happens

We have built a random demand generator for the 3 years that assumes independent demand (0 correlation) from year to year



Give it a try!

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Check with your neighbors...

Check the solution sheet...

Ask me questions...



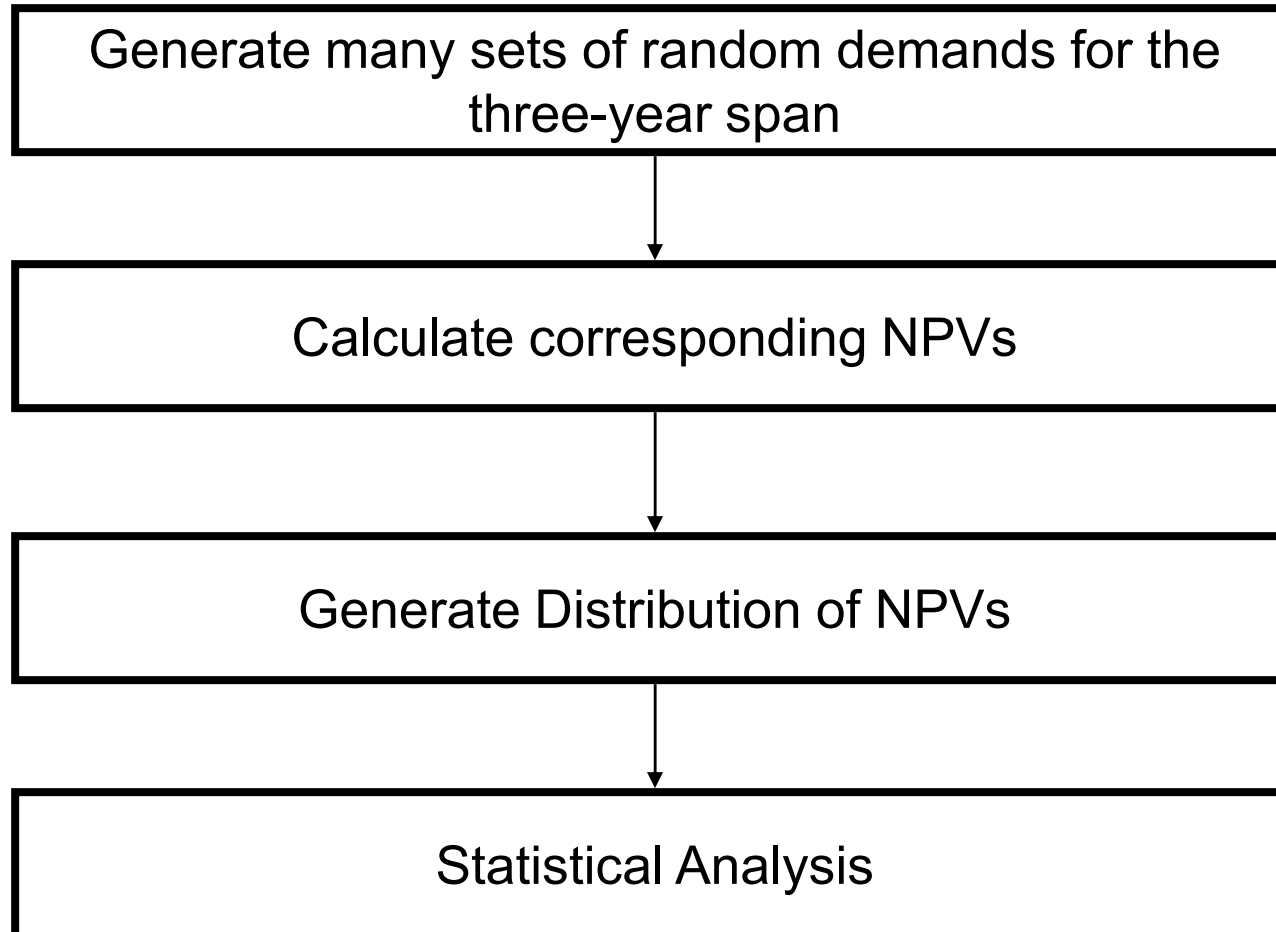
# How Monte Carlo Simulation works

Calculate two  $NPV_A$ s corresponding to the two random demand simulations

Demand in Year 1	Demand in Year 2	Demand in Year 3	$NPV_A$
345	678	1001	?
189	579	690	?

How about generating many sets of random demands, and get the corresponding  $NPV_A$ s automatically?

# Monte Carlo Simulation



# Setup simulation by Data Table

Follow these instructions, step by step:

1. Link demand in sheet for Plan A to the random demand generator, specifically, Plan A!E5 = Rand!C3; Plan A!G5 = Rand!D3; Plan A!I5 = Rand!E5
2. In "Simulation" sheet, type "='Plan A'!C16" in cell B8 ("='Plan A'!C16" is the output of result for  $NPV_A$ )
3. Create the Data Table. Select "A8:B2008", click "Table" under "Data" menu, in "column input cell" put "A7", leave "row input cell" blank.
4. Same thing already done for Plan B

NOTE: there is no input in the value column of the Data Table; an empty cell is selected as the "column input cell". Why?

# Explanation

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- For the One-Way Data Table, there is no need to set up the input values in a list, since each row of the Data Table calls RAND() and generates an NPV<sub>A</sub> projection
- We have 2,000 rows in the Data Table, so we have simulated 2,000 times
- Click “command =” or “F9” to try another simulation run



Give it a try!

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# Calculating descriptive statistics

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- Useful to know  $E[\text{NPV}]$ , maximum, and minimum values for the simulated results

Follow step by step:

1. In Cell D1 type `"=AVERAGE(B$9:B$2008)"`
2. In Cell D2 type `"=MAX(B$9:B$2008)"`
3. In Cell D3 type `"=MIN(B$9:B$2008)"`



Give it a try!

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Ask me questions...

# Deterministic vs. dynamic results

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- From the base case spreadsheet, we learn  $NPV_A = \$162.1\text{M}$  and  $NPV_B = \$156.5\text{M}$
- What is your result for the  $E[NPV_A]$  and  $E[NPV_B]$  when considering demand uncertainty?
- Jensen's inequality and the Flaw of Averages:

$$f[E(x)] \neq E[f(x)]$$



# Target curve

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- The target curve is another name for cumulative distribution function (CDF)
- In our case, a target curve aims at making a representation to managers that
  - “There is a probability  $X$  that NPV will be lower (higher) than a **targeted**  $Y$  dollars for this project”
- Value At Risk is a common language on Wall Street. It stresses downside risk, though we should also look at CDF for upside potential of a project, or Value At Gain!

# Target curve

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Follow the instructions, step by step:

1. In sheet "Simulation", set Cell G7 " $=\$D\$3+(\$D\$2-\$D\$3)/20 * F7$ ", and drag the formula down to G27
2. Set Cell H7 " $=\text{COUNTIF}(\$B\$9:\$B\$2008, "<=" \& G7)$ ", and drag the formula down to H27
3. Set Cell I7 " $=H7/2000$ ", and drag down to cell I27
4. Same is already done for Plan B

# Target curve

6. Right-click the chart on the right, select "Source Data"
7. Select "Series", and press "Add". This adds a new data series to the graph. Call it "NPV<sub>A</sub>"
8. Select the range =Simulation!\$G\$7:\$G\$27 for X values, and the range =Simulation!\$I\$7:\$I\$27 for Y values. Click "OK"
9. Right-click the curve and change "Weight" to 3
10. Hit "command =" or "F9" and watch the target curve move !

# Explanation

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- We set up 20 data buckets and count how many data points fall into each interval
- “=COUNTIF()” function counts the number of cells within a range that meet the criteria
- The Excel file demonstrates how you can:
  - Add  $E[NPV_A]$  and  $E[NPV_B]$  as vertical lines
  - Add histograms for two NPV distributions using the information created earlier
- Can also use the Histogram analysis tool in “Data Analysis” package, but it won’t refresh

# Values At Risk and Gain

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- Use your cursor on the graph to find different Values At Risk and Values At Gain
- Alternatively, use the percentile function
  - In cell N5, type 10%
  - In cell R5, type  
“=PERCENTILE(B9:B2008,N5)”
- What does this tell you?
- That’s interesting information for managers and decision-makers!

# Question

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- Why are high NPV values more cut off for Plan B on the target curve and histogram than for Plan A?
  - A matter of constraints...



Give it a try!

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Check with your neighbors...

Check the solution sheet...

Ask me questions...

# Next class...

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- Today's session modeled demand uncertainty based on a uniformly distributed random variable
- This is not necessarily realistic, though it is simple and sufficient for today's purposes
- Next session explores alternative probability distributions from which to sample and stochastic models
- **STAY TUNED!**